

الدالة اللوغاريتمية [I]

Fonction logarithme

تمرين 1

عَيِّن مجموعة تعريف الدالة f حيث:

$$f(x) = x - \ln(2x - 3) \quad (1)$$

$$f(x) = x - 1 + \ln|2 - x| \quad (2)$$

$$f(x) = \frac{x}{x+1} - \ln x^2 \quad (3)$$

$$f(x) = \frac{2x}{2x-1} - \ln \sqrt{x+1} \quad (4)$$

$$f(x) = \frac{1}{x} - \ln(x^2 - 4) \quad (5)$$

$$f(x) = x + 1 + \ln(x - 4)^2 \quad (6)$$

$$f(x) = \ln(-x^2 - x + 2) \quad (7)$$

$$f(x) = 2x - 1 + \ln\left(\frac{x}{x-1}\right) \quad (8)$$

$$f(x) = \ln x + \ln(x - 1) \quad (9)$$

$$f(x) = \frac{1}{x} + \ln\left(\frac{2+x}{2-x}\right) \quad (10)$$

$$f(x) = 2x + 3 + \ln\left|1 + \frac{3}{x}\right| \quad (11)$$

$$f(x) = \frac{x}{\ln(x+1)} \quad (12)$$

$$f(x) = \sqrt{1 - \ln x} - \ln(x^2 + 1) \quad (13)$$

$$f(x) = \frac{1 + \ln x}{1 - |\ln x|} \quad (14)$$

تمرين 2

حل في المجموعة $\mathbb{R}$ المعادلات التالية:

$$\ln(2x - 3) = \ln(x - 3) + \ln 5 \quad (1)$$

$$\ln(x^2 - 5) - \ln(4 - x) = 2 \ln 2 \quad (2)$$

$$\ln|x - 4| + \ln(7 - 3x) = \ln 2 \quad (3)$$

$$\ln x^2 - \ln \sqrt{x} - 6 = 0 \quad (4)$$

$$(\ln x)^2 - \frac{5}{2} \ln x + 1 = 0 \quad (5)$$

$$(\ln x)^4 - 10(\ln x)^2 + 9 = 0 \quad (6)$$

$$(\ln x)^3 + 3(\ln x)^2 - 4 = 0 \quad (7)$$

$$\ln(\sin x) + \ln(\cos x) = \ln \frac{\sqrt{3}}{4} \quad (8)$$

تمرين 3

حل في $\mathbb{R}^2$ جمل المعادلات التالية:

$$\begin{cases} \ln x + \ln y = \ln 10 \\ x + y = 7 \end{cases} \quad (1)$$

$$\begin{cases} \ln(x+1) + 2\ln(y-2) = 4 \\ 3\ln(x+1) - \ln(y-2) = 5 \end{cases} \quad (2)$$

$$\begin{cases} \ln x + \ln y = 1 \\ \frac{3}{\ln x} - \frac{2}{\ln y} = 2 \end{cases} \quad (3)$$

$$\begin{aligned} & (e^2 - 1; e + 2) \quad (5; 2), (2; 5) \\ & (\sqrt{e}; \sqrt{e}), (e^3; e^2) \end{aligned}$$

تمرين 4

عَيِّن الدالة المشتقة للدالة f في المجموعة التي تكون فيها قابلة للاشتقاق:

$$f(x) = \ln(x^2 + 4x - 5) \quad (1)$$

$$f(x) = 2x - 1 + \ln|2x - 1| \quad (2)$$

$$f(x) = \ln(-x) + \ln \sqrt{2x + 3} \quad (3)$$

$$f(x) = \frac{1 + 2 \ln x}{x + 1} \quad (4)$$

$$f(x) = \frac{1 + \ln x^2}{x + 1} \quad (5)$$

$$f(x) = x \ln x - \ln(\ln x) \quad (6)$$

$$f(x) = x + \ln\left(\frac{2x-1}{2x+1}\right) \quad (7)$$

$$f(x) = \sqrt{\ln x} + (\ln x)^2 \quad (8)$$

تمرين 5

احسب النهايات التالية:

$$\lim_{x \rightarrow +\infty} [x + \ln(2x + 3)] \quad (1)$$

$$\lim_{x \rightarrow -\infty} \ln(x^2 + x - 2) \quad (2)$$

$$\lim_{x \rightarrow 0^+} \frac{1 + \ln x}{x^2} \quad (3)$$

$$\lim_{x \rightarrow +\infty} (x \ln x - x) \quad (4)$$

$$(X = -x) \lim_{x \rightarrow -\infty} [2x - 1 + \ln(-x)] \quad (5)$$

$$\lim_{x \rightarrow +\infty} \frac{x \ln x}{x + 1} \quad (6)$$

$$\lim_{x \rightarrow 0^+} \left(\frac{x+1}{x} + \ln x \right) \quad (7)$$

$$\lim_{x \rightarrow +\infty} \ln\left(1 + \frac{1}{x}\right) \quad (8)$$

$$\lim_{x \rightarrow -\infty} \left[x + \ln\left(\frac{x}{x-2}\right) \right] \quad (9)$$

$$\lim_{x \rightarrow 2} \left[x + \ln\left(\frac{x-2}{x+2}\right) \right] \quad (10)$$

$$\lim_{x \rightarrow 2} \left[x + \ln\left(\frac{2+x}{2-x}\right) \right] \quad (11)$$

$$\lim_{x \rightarrow -\infty} \ln \left| \frac{x+1}{x^2-4} \right| \quad (12)$$

$$\lim_{x \rightarrow +\infty} \frac{\ln(x+2)}{x} \quad (13)$$

$$\lim_{x \rightarrow +\infty} \frac{\ln(x^2+3)}{x} \quad (14)$$

$$\lim_{x \rightarrow +\infty} \frac{\ln(x^2+1)}{x^2} \quad (15)$$

$$\lim_{x \rightarrow +\infty} \frac{\ln x}{(x+1)^2} \quad (16)$$

$$\lim_{x \rightarrow 0^+} (x \ln x^2 - x) \quad (17)$$

$$\lim_{x \rightarrow 0^+} (\sqrt{x} \ln x - \sqrt{x}) \quad (18)$$

$$\lim_{x \rightarrow 1} (1-x) \ln(1-x) \quad (19)$$

$$\lim_{x \rightarrow +\infty} [\ln(x^2-1) - \ln(x+2)] \quad (20)$$

$$\lim_{x \rightarrow +\infty} [x^2 - 2 - \ln(x-1)] \quad (21)$$

$$\lim_{x \rightarrow +\infty} \frac{2x - \ln x}{x + \ln x} \quad (22)$$

$$\lim_{x \rightarrow 0^+} \frac{\ln(x+1)}{x^2} \quad (23)$$

$$\lim_{x \rightarrow 0^+} \frac{\ln(2x+1)}{x} \quad (24)$$

$$\lim_{x \rightarrow 2} \frac{\ln(x-1)}{x-2} \quad (25)$$

$$(X = \frac{1}{x}) \lim_{x \rightarrow +\infty} x \ln\left(1 + \frac{1}{x}\right) \quad (26)$$

$$\lim_{x \rightarrow 0^+} x \ln\left(1 + \frac{1}{x}\right) \quad (27)$$

ادرس تغيرات كل دالة ثم ارسم بيانها:

$$]-1; +\infty[\quad f(x) = x^2 - 5x + 5\ln(x+1) \quad (1)$$

$$]0; +\infty[\quad f(x) = \ln x + \frac{1}{x} - 1 \quad (2)$$

$$]-\infty; 0[\cup]0; +\infty[\quad f(x) = \frac{x-1}{x} + \ln|x| \quad (3)$$

$$]-1; +\infty[\quad f(x) = \frac{x}{x+1} + \ln(x+1) \quad (4)$$

$$]-\infty; +\infty[\quad f(x) = \ln(x^2 - 4x + 5) \quad (5)$$

$$\mathbb{R} - \{-2; 2\} \quad f(x) = \ln|x^2 - 4| \quad (6)$$

$$]-1; +\infty[\quad f(x) = -x + 1 + \ln(x+1) \quad (7)$$

$$]-\infty; 0[\cup]0; +\infty[\quad f(x) = 2x - \ln x^2 \quad (8)$$

$$]0; +\infty[\quad f(x) = x \ln x - x \quad (9)$$

$$]0; +\infty[\quad f(x) = \frac{2+3\ln x}{x} \quad (10)$$

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$$]0; 3[\quad f(x) = \ln\left(\frac{x}{3-x}\right) \quad (12)$$

$$]-\infty; -1[\cup]0; +\infty[\quad f(x) = x + 2\ln\left(1 + \frac{1}{x}\right) \quad (13)$$

$$\mathbb{R} - \{-2; 2\} \quad f(x) = -x + \ln\left|\frac{x+2}{x-2}\right| \quad (14)$$

$$]0; +\infty[\quad f(x) = (\ln x)^2 + \ln x - 2 \quad (15)$$

$$]0; \sqrt{e}[\cup]\sqrt{e}; +\infty[\quad f(x) = \frac{\ln x}{2\ln x - 1} \quad (16)$$

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